Mendel
University
in Brno

MENDELU Working Papers
in Business and Economics

10/2011

A stochastic programming model of the sowing plan with crop succession restrictions

Jitka Janová

MENDELU Working Papers in Business and Economics
Research Centre
Faculty of Business and Economics
Mendel University in Brno
Zemědělská 1, 613 00 Brno
Czech Republic
http://vyzc.pef.mendelu.cz/en
+420 545 132 605

Citation

Janová, J. (2011). A stochastic programming model of the sowing plan with crop succession restrictions. *MENDELU Working Papers in Business and Economics* 10/2011. Mendel University in Brno. Cited from: http://vyzc.pef.mendelu.cz/cz/publ/papers

Abstract

Jitka Janová: **A stochastic programming model of the sowing plan with crop succession restrictions**

A user-friendly decision support model for agricultural production planning in the Czech Republic is developed covering both the randomness of harvest parameters entering the decision problem and the complex crop succession requirements. The methods of stochastic programming are applied and the linear re-sowing constraints are developed, described in detail and incorporated in such a way that the model can be approached by software tools commonly available at farms. The model abilities are demonstrated in the particular case of production planning decision making in the South Moravian farm. The re-sowing constraints themselves are verified with respect to covering the local crop succession requirements and the validation of the model as a whole is performed using a Monte Carlo simulation.

Key words

stochastic programming, crop rotation, crop plan, agriculture optimization

JEL: C44, C61, Q15

Contacts

J. Janová, Department of Statistics and Operational Research, Faculty of Business and Economics, Mendel University in Brno, Zemědělská 1, 613 00 Brno, Czech Republic, tel.: +420 545 132 400, e-mail: janova@mendelu.cz

Acknowledgements

This work was supported by the grant MSM6215648904 of Czech Ministry of Education, Youth and Sport of Czech Republic

Introduction

Since the beginning of operations research, the optimization tools have extensively been used for developing the agriculture production planning decision support tools (for recent contribution see e.g. [17]). The optimization problems in agricultural sciences are more difficult to solve by common optimization techniques than similar problems in other areas due to the complexity of decision making that has to take into account the environmental aspects. Typically, the problems deal with uncertainty and, therefore, the formulated mathematical programming models should be approached by the methods of stochastic programming. The stochastic programming approach to production planning in agriculture is currently under worldwide research and some models solving particular stochastic optimization problems were developed (see e.g. [3], [2], [15], [22]).

Other source of complexity in agriculture production planning follows from the crop succession requirements which are not easy to incorporate into mathematical programming models. Currently, there is a running research on the inclusion of the crop rotation restrictions into mathematical programming models of production planning in agriculture. In [11], the method of writing the crop succession requirements as linear programming constraints in an LP-based model for agricultural production planning is presented and in [5] the dynamical programming approach to crop rotation modeling is described. Various approaches are used for developing the local agricultural decision support models considering crop rotation (see [6] for network flow, [14], [19] for linear programming [18],[2] for stochastic programming and [20] for dynamic control approach). Some software tools supporting the agricultural decision making concerning crop rotations were developed (see [7], [1]).

The randomness of the key parameters of the optimization problem and the need for following the rotation rules as key features of the production planning optimization problem should enter the related mathematical programming model to serve as a decision making support tool. However, a combination of these two aspects in a single model is overall rare (for a similar problem in potato research see [16]). Hence, none of the results of the current research in the field of agriculture production planning is immediately applicable to problems comprising both of the above mentioned features. Moreover, the models developed so far are rather complex and not easily solvable by common SW tools.

Generally, farmers in the Czech Republic decide about the sowing plan for the next period using their expert knowledge and the past experience. Apart from the rotation rules, their decision making is restricted by many other factors such as cost, labor, manure, or land availability. Other restrictions arise due to more strict ecological rules being incorporated into the agricultural business in the Czech Republic concerning water and soil preservation, etc. These new restrictions will make the decision making in agriculture production planning more complex and the results obtained by intuitive approaches may considerably differ from the optimal solution. Therefore, the decision support systems - not being used so far - are becoming more important for the Czech farmers.

The aim of this paper is to develop an agriculture production planning decision making support model covering the stochastic nature of the harvest parameters entering the problem and incorporating the crop succession requirements into the model as a set of linear re-sowing constraints in such a way that the resulting model can be processed by EXCEL. Such an approach offers the farmers a mathematical-programming-based user-friendly alternative to estimative decision making used so far. The model will be developed using the real situation of typical representative farm.

In Sec. 2 the stochastic programming model of a farmer's production planning is formulated and in Sec. 3 detailed description of the re-sowing constraint construction is provided. The model developed is applied to a particular problem of the South Moravian farm and the model properties are demonstrated and discussed in Sec. 4. Finally, in Sec. 5 the re-sowing constraint is verified with respect to covering the local crop succession requirements and the validation of the model as a whole is carried out using the Monte Carlo simulation.

Stochastic programming approach to farm production planning

The farmer's objective is to maximize profit from the future harvest for a given area of arable land. The aim is to find total areas of land to be cropped by particular crop plants. We consider a representative typical farm in the Czech republic managing a single compact one-soil type area of land. Note that about 70% of total arable land is controlled by agricultural cooperatives farming on areas of up to 1500 ha in the Czech Republic with the areas mostly undivided and compact, hence, due to the soil conditions in the Czech Republic, very often the soil at a farm can be considered to be of a single type. If not, it can be divided into several areas of one soil type for each of which the presented model can be applied. As typical for these farms, we will assume associated animal production generating the requirements on a certain level of feed crops and limiting the manure available. We assume that the crop succession rules must be obeyed and the farmer uses his own capital.

Let us consider first the general production planning problem without crop rotation restrictions. The farm maximizes its profit from the production under restrictions on land, manure and capital and also specific restrictions resulting from the associated animal production may arise. This optimization task can be formulated as a mathematical programming problem:

$$z^\star = \max \ \Sigma_{i=1}^{n} c_i x_i \tag{1}$$

$$\sum_{i=1}^{n} a_{ij} x_i \leq b_j \tag{2}$$

$$q_i x_i \geq Q_i \tag{3}$$

$$x_i \geq 0,\ 1 \leq i \leq n. \tag{4}$$

where the decision variables x_i stand for areas of arable land planted with crop i. In the objective function (1) the parameters c_i are the random variables of *total profit* per 1 ha of planted crop i defined as follows:

$$c_i = p_i q_i - n_i, \tag{5}$$

q_i being the random variable *yield* of the respective crop-plant i. We consider n_i (*total costs* per 1 ha of arable land planted by crop i) and p_i (*selling price* for 1 ton of crop i), $1 \leq i \leq n$, constants of known values. Apart from standard inequality constraints (2) where a_{ij}, b_j are known constants, the model must cover also the restrictions on minimal feasible harvests of particular crops (3).

The formulation (1–4) seems to represent a linear programming model but, as the harvests q_i must be seen as random variables, the problem appears to be of a stochastic nature. Note that, through (5), the profits c_i are random as well. The prices per ton p_i change from one harvest to another, but for the purpose of this model we assume that, for the next period, the prices are already given (their respective values are obtained by the expert estimation of the farm management).

We will employ the Markowitz criterion to obtain a deterministic equivalent of the objective function in the stochastic programming problem (1–4). Note that there is a number of approaches to the transformation of a stochastic program into a deterministic one. The Markowitz model was one of the first approaches used in the agriculture production planning optimization under risk (see [9]). The main idea is to maximize the expected return and simultaneously minimize the variability of return. Following [9], we assume the yields and unit profits being normally distributed random variables $q_i \sim N(\mu_i, \sigma_i^2)$, $c_i \sim N(\gamma_i, \tilde{\sigma}_i^2)$. Denoting

Σ the covariance matrix of the random vector $(c_1, \ldots, c_n)$

x the decision variables vector,

a the risk aversion coefficient,

the Markowitz-type objective function takes the form

$$z^\star = \min \left(\frac{a}{2} x \Sigma x^T - \gamma x^T \right),$$

where the maximization of the random profit (1) is replaced by the minimization of the difference between the terms representing the variability and the mean value of total profit. Further, the constraint (3) where the random parameters q_i appear must be replaced by a deterministic constraint. Commonly, this is done through a requirement that the respective restrictions should be satisfied at least with probability π (see [8]). This requirement can be written as

$$P\{Q_i - q_i x_i \leq 0\} \geq \pi,$$

which, after standardization, can be rewritten as

$$F\left(\frac{\mu_i x_i - Q_i}{\sigma_i x_i}\right) \geq \pi,$$

where F denotes the cumulative distribution function of the standard normal distribution. Finally, we obtain a deterministic constraint in the form

$$\left(\mu_i - F^{-1}(\pi)\sigma_i\right) \cdot x_i \geq Q_i.$$

This means that it is required that the area sown by a certain crop should be greater than the area where the expected value of harvests is equal to the required value. Note that the probability π must be set to a reasonable level for the constraint to produce feasible and realistic area limits. The initial stochastic programming problem (1–4) is now transformed into a deterministic quadratic programming problem:

$$z^\star = \min \left(\frac{a}{2} x \Sigma x^T - \gamma x^T \right) \tag{6}$$

$$\sum_{i=1}^{n} a_{ij} x_i \leq b_j \tag{7}$$

$$\left(\mu_i - F^{-1}(\pi)\sigma_i\right) \cdot x_i \geq Q_i \tag{8}$$

$$x_i \geq 0,\ 1 \leq i \leq n. \tag{9}$$

The crop re-sowing constraints

The crop re-sowing constraints ensure that the same crop will not be re-sowed on one piece of land during relevant number of years and that the prohibited succession of two different crops will not appear. These restrictions can be represented by a system of linear inequalities. Denote n the number of crop types planted by a particular farm, and x_i, $1 \leq i \leq n$ the area planted by crop i. Then the constraints are generated using the algorithm (for simplified version see [12])

for p =1 to n:
    for each p-combination $\{ i_1, \ldots, i_p\}$ from a set $\{ 1, \ldots, n \}$:

$$\sum_{s=1}^{p} x_{i_s} \leq X - \tilde{x}_{i_1 \ldots i_p} - \tilde{y}_{i_1 \ldots i_p} \tag{10}$$

where X is the total area of arable land available, $\tilde{x}_{i_1 \ldots i_p}$ is the total area cropped with all the crops $i_1 \ldots i_p$ during the $r(i_1), \ldots, r(i_p)$ past years, $r(i_s)$ is the number of years after which the crop i_s

Table 1: Crop design history

field	A	B	C	D
2 years ago	1	2	4	2
1 year ago	2	4	2	3

Table 2: The values of $\tilde{x}_{i_1}$ $\tilde{y}_{i_1}$ for particular crops

crop	1	2	3	4
$\tilde{x}_{i_1}$ [ha]	1	2	0	0
$\tilde{y}_{i_1}$ [ha]	1	1	0	0

may be planted on the same area, $\tilde{y}_{i_1 \dots i_p}$ is the total area not cropped with all the crops $i_1, \dots, i_p$ during relevant past years, but cropped last year with the crop plants after which none of the crops $i_1, \dots, i_p$ may succeed.

Let us perform the construction of the resowing constraint using a small-scale example. Let us have only four types of crops (i.e. $n = 4$). Crop 1 may be resowed after 2 years (i.e. $r(1) = 2$) on the same piece of land, while crop 2 after 1 year (i.e. $r(2) = 1$) and both of them must not directly succeed crop 3. Other two crop types may be resowed without restrictions (i.e. $r(3) = r(4) = 0$). Let us have 4 fields (A, B, C, D) each of them of 1 ha area and suppose there is a 2 years history of the crop design for each field known (see table 1).
For $p = 1$ we get the first set of conditions:

$$x_{i_1} \leq X - \tilde{x}_{i_1} - \tilde{y}_{i_1}, \tag{11}$$

where on the left-hand side, there is the total area of land sowed by the crop i_1, and on the right-hand side, there is the total area of the arable land available less the area, where the crop i_1 was sowed during past $r(i_1)$ years and less the area not already covered by $\tilde{x}_{i_i}$, where the crop, that must not precede to crop i_1 was sowed last year. Hence, taking into account that i_1 gradually takes all the combinations of the indexes $1, 2, 3, 4$, the condition (11) represents four constraints, each of them for one of the crops planted in the farm. These constraints can be written down enumerating the values of the prohibited areas $\tilde{x}_{i_1}$ $\tilde{y}_{i_1}$ from the crop history (see table 2). The constraints 11 take the form:

$$x_1 \leq 2,\ x_2 \leq 1,\ x_3 \leq 4,\ x_4 \leq 4. \tag{12}$$

If no other constraints entered the problem, the solution $x_1 = 2$, $x_2 = 1$ would be feasible. However, sowing crop 1 on the fields B and C there is no feasible field for planting crop 2. Therefore, another set of constraints must emerge in the problem.

Indeed, for $p = 2$, restriction (10) leads to

$$x_{i_l} + x_{i_2} \leq X - \tilde{x}_{i_1 i_2} - \tilde{y}_{i_1 i_2}. \tag{13}$$

On the left-hand side, there is the total area of the arable land sowed by crop i_1 or crop i_2. On the right-hand side, there is the total area available less the area, where both of the crops occurred during past relevant numbers of years and less the area not included in $\tilde{x}_{i_1 i_2}$, where the crops after which both crops i_1 and i_2 must not succeed were sowed last year. In (13), coefficients (i_1, i_2)

range over all the combinations of size 2 from the set $\{1, 2, \ldots, n\}$. Hence, (13) forms a set of conditions for all pairs of crops planted:

$$x_1 + x_2 \leq 2 \quad (14)$$
$$x_1 + x_3 \leq 4 \quad (15)$$
$$\vdots$$

The total area cropped by plants 1 and 2 is decreased in (14) by 1 ha were both plants were cropped during relevant number of years and by another 1 ha on which plant 3 was planted last year.The other constraints arising for $p = 2$ and all for $p = 3$ need not be written, because crops 3 and 4 need not meet any succession requirements and all of the possibly arising constraints are included in (16) as obtained for $p = 4$:

$$x_1 + x_2 + x_3 + x_4 \leq 4. \quad (16)$$

Hence, the case of sowing more than one crop at the same time in the same area gets prevented. Although it seems that there will be a large number of conditions in the model and handling the model will be complicated, thanks to the low real number of both the crops planted on farms and number of years during which the crops cannot be resowed in the same area, the volume of the model remains reasonable.

Planning the production: The South Moravian farm case

In this section, we consider a typical agricultural production unit in Czech Republic: the agriculture cooperative farms in the South Moravian region on the area of 1325 ha. Due to the favorable weather and natural conditions in the Czech Republic, the farmer is not bothered by irrigation or other worldwide problems and the sowing plan decision making is restricted simply by constraints on

- total area of arable land available (X),
- capital (N),
- maximal area fertilized by manures (M),
- minimum volume of feed crops (Q_i, i, $1 \leq i \leq n$),
- minimal resp. maximal area cropped by particular crop (A_i resp. B_i, i, $1 \leq i \leq n$),
- re-sowing of the crops.

Note, that also many other constraints may be taken into account depending on a particular farmer, the restrictions listed above represent the set of restrictions typical for the agriculture business in Czech Republic. We assume that there is only one soil type in the whole area of arable land of a particular farmer. Hence, all the crops may be planted on any part of the arable land. The farmer uses his own capital, which must cover the total costs of planting all the crops. The total costs C_i of planting crop i consists of the costs of seeds, labor and machine time used for planting the particular crop. The constraints on the maximal area fertilized by manure and the thresholds for the areas cropped by particular crops follow from the livestock breeding potential and needs at the particular farm. Finally, the crop re-sowing constraints stem from the fact, that a succession of certain crops on the same piece of land is not allowed or not advisable, otherwise soil fertility will decrease or the crops may become susceptible to diseases.

The model solution: process and results

A particular case of planning the production for the next period in a South Moravian agriculture cooperative is considered. The data were gathered in [13], where a linear programming problem without rotation restrictions was solved. The decision variables x_i are defined as the areas planted by particular crops (see Tab. 3). Due to different prices of the food and feed crops, each area sowed by one agricultural crop was split into two decision variables. Nevertheless, the harvest characteristics are for both food and feed type of the crop the same.The quadratic programming model discussed in Sec. 2 takes, for the particular case of a South Moravian farm, the form

$$z^\star = \min \left(\frac{a}{2}x\Sigma x^T - \gamma x^T\right) \tag{17}$$

$$\sum_{i=1}^{n} n_i x_i \leq N \tag{18}$$

$$\sum_{i=1}^{n} m_i x_i \leq M \tag{19}$$

$$\left(\mu_i - F^{-1}(\pi)\sigma_i\right) \cdot x_i \geq Q_i \tag{20}$$

$$x_i \geq A_i \tag{21}$$

$$x_i \leq B_i \tag{22}$$

$$p = 1 \text{ to } n :$$

$$\text{for each } p - \text{combination } \{i_1, \ldots, i_p\} \text{ from} \{1, \ldots, n\} :$$

$$\sum_{s=1}^{p} x_{i_s} \leq X - \tilde{x}_{i_1 \ldots i_p} - \tilde{y}_{i_1 \ldots i_p} \tag{23}$$

$$x_i \geq 0,\ 1 \leq i \leq n. \tag{24}$$

The restrictions on the total capital available are contained in (18) where n_i are unit costs and N denotes the budget. The constraint (19) reflects the restriction on the total manure available in a simplified form. The amount of manure used on each field depends on both the plant to be cropped and the preceding production on the field. To keep the model simple, we quantify in (19) the approximate rule used by the farmer himself in routine decision making about crop plan. The maximal area that can be fertilized by manure is $M = 400$ ha (this corresponds to the amount of manure available and the average amount of manure used per 1 ha). Based on experience, the farmer states, that, considering this "average fertilization", the corn area is commonly fertilized on 2/3 of acreage, spring barley and oilseed rape on 1/5 of acreage and potatoes on the total acreage (these coefficients are denoted m_i in constraint (19)). The restriction (20) ensures that the expected yield of feed crops will be at least at the minimum acceptable level and conditions (21) and (22) ensure the maximal and minimal areas cropped by particular crops and (23) are the crop resowing constraints developed in Section 3.

The parameters of the constraints are listed in Tab. 4.

Identifying the minimum number of years after which each crop can be resowed on the same area and the prohibited succession of crops (see e.g. [23]), the set of resowing constraints (23) is generated:

$$\begin{aligned} \Sigma_{k=1}^{9} x_k &\leq 979, \\ x_{12} &\leq 131, \\ x_{12} + \Sigma_{k=1}^{9} x_k &\leq 1125, \end{aligned} \tag{25}$$

Note that, for the purpose of generating the constraint, crops $1-9$ were clustered together as cereals. The model was solved for the circumstances of the year 2009, for $\pi = 0.75$, which reflects

Table 3: Decision variables

x_i	area sowed by
x_1	winter food wheat
x_2	winter feed wheat
x_3	spring food wheat
x_4	spring feed wheat
x_5	winter food barely
x_6	winter feed barely
x_7	spring food barely
x_8	spring feed barely
x_9	triticale
x_{10}	corn
x_{11}	corn silane
x_{12}	oilseed rape
x_{13}	potatoe
x_{14}	grass
x_{15}	grass seed

Table 4: The parameters of the problem

i	γ_i [CZK/ha]	m_i	Q_i [t]	n_i [CZK/t]	p_i [CZK/ha]	μ_i	σ_i^2
1	2590,0	0	-	20524	3900	5,93	0,46
2	1373,3	0	380	17592	3200	5,93	0,46
3	1764,5	0	-	15428	3900	4,41	0,93
4	882,7	0	70	13224	3200	4,41	0,93
5	5513,9	0	-	16088	5300	4,08	0.82
6	1384,3	0	70	12066	3300	4,08	0.82
7	5570,5	0,2	-	17776	5300	4,41	0.41
8	1204,5	0,2	300	13332	3300	4,41	0.41
9	433,7	0	-	15457	3200	4,97	0,57
10	1856,0	0,666667	-	33408	4800	6,19	8.43
11	1726,5	0,666667	7900	15906	600	29,39	38,72
12	3185,3	0,2	-	24256	8800	3,12	0,35
13	1397,1	1	-	64589	2740	24,08	19,97
14	-561,5	0	-	12933	485	25,51	13,16
15	3041,3	0	-	8420	26500	0,43	0,01

Table 5: The optimization results for $\pi = 0.75$ and different settings of risk aversion parameter a

	$\mathbf{a}\cdot 10^6$	0-0.5	0.6	0.7	0.8	0.9	1.0	decision made
x_1	winter food wheat	277	272	256	223	194	183	188
x_2	winter feed wheat	69	69	69	69	69	69	78
x_3	spring food wheat	0	4.5	11	8	0	0	0
x_4	spring feed wheat	17	17	17	17	24	36	13
x_5	winter food barley	2	2	2	2	2	2	0
x_6	winter feed barley	20	20	20	20	20	20	17
x_7	spring food barley	254	254	254	254	254	254	260
x_8	spring feed barley	76	76	76	76	76	76	74
x_9	triticale	0	0	9	40	40	40	0
x_{10}	corn	20	20	20	20	20	20	29
x_{11}	corn silage	314	314	314	314	314	314	230
x_{12}	oilseed rape	131	131	131	131	126	122	230
x_{13}	potatoes	0	0	0	0	0	2	0
x_{14}	grass	60	60	60	65	100	100	79
x_{15}	grass seed	85	85	85	85	85	85	64

the real farmer's attitude to keeping the constraint (3) and for several values of the risk aversion coefficient $a = 0, 0.1 \cdot 10^{-6}, \dots, 1 \cdot 10^{-6}$ covering the low aversion to risk with $a = 0$ up to high risk aversion attitude of the decision maker with $a = 1 \cdot 10^{-6}$. The results of the optimization model for all choices of parameter a can be compared to the real decision of the farmer in Tab 5. The differences between the model results and the real decision are not great except area cropped by oilseed rape. In the optimized model, the oil seed rape area (122 ha –131 ha) is much smaller than the one in the real plan of the cooperative (230 ha). The resowing constraint allows cropping only 131 ha of land by oilseed rape, but the resulting area is, for higher risk aversion coefficient, even smaller due to the variability in oilseed rape harvests. Hence, for the year 2009, the farmer broke the crop succession restrictions and his solution is infeasible by the model. In Sec. 5.2, we will compare the expected profit for the optimal plans obtained by the model with the farmer's decision.

Solving the model in Excel

The quadratic programming problem (17-24) can be solved using any available optimization software including the Solver in Excel. The standard Excel Solver has a limit of 200 decision variables or changing cells. It also imposes a limit on the number of constraints in a situation where the problem is nonlinear (there is a limit of 100 constraints other than constant bounds on the variables and integer constraints). Since, in our quadratic programming problem, there are 15 decision variables and about 50 constraints including constant bounds, Solver appears to be an appropriate software tool for finding the solution. Note that the covariance matrix and mean values needed as parameters in the model can easily be enumerated with the use of Excel. Also the right hand sides of the re-sowing constraints (23) (quantified for the particular case study in (25)) can be determined with the help of Excel, but one must be aware of the fact that all the crop history on each field must be entered into the spreadsheet table. In the event that there is no electronic version of the crop history at a farm, it may appear more simple to determine the left hand sides of re-sowing constraints by hand.

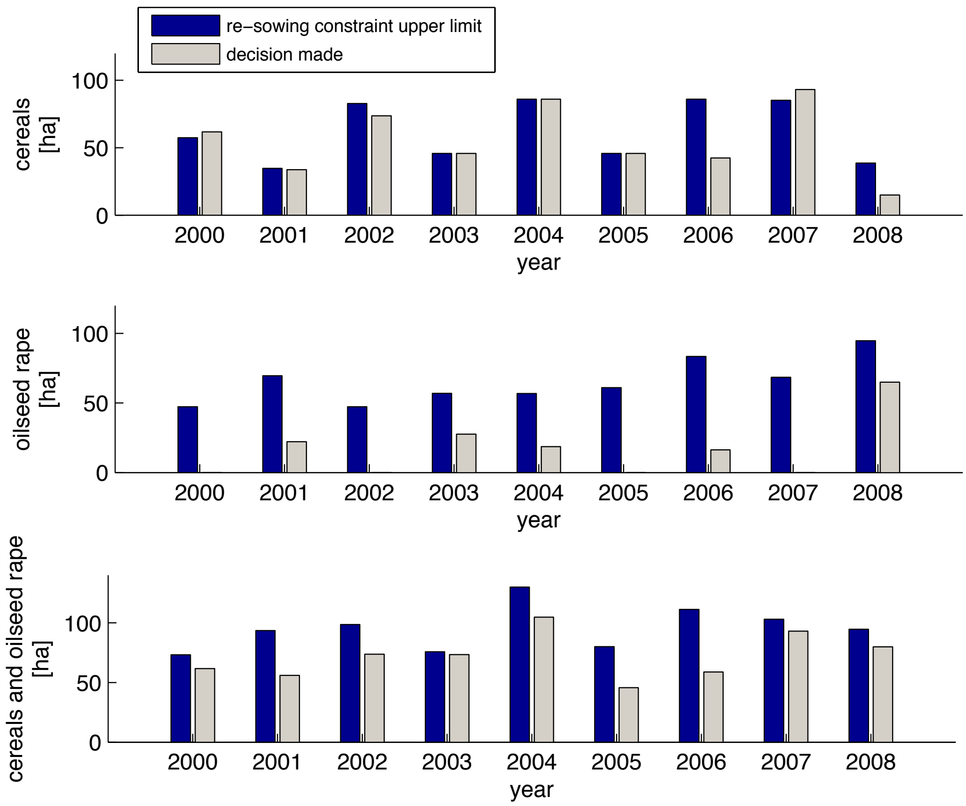

Figure 1: Arable land for cereals and oilseed rape: the re-sowing constraint limits compared to real decision (period 2000-2008).

Verification and validation of the model

The model aiming to serve as a decision support tool must be technically correct and valid in the sense of fulfilling the goals for which it was created (see [10]). In our optimization problem, we claim the practical feasibility of the solution (i.e. all relevant and important constraints must be fulfilled) and the maximal profitability of the solution. We assume that the model (17-22) is verified since it is a standard approach to crop plan optimization problem developed by Freund [9] and later on applied by many authors to agricultural optimization problems (see e.g. [4]). Only the re-sowing constraint as a modification of the well known model should be verified. This constraint ensures that the optimal sowing plan meets the crop succession requirements. The verification will be done by comparing the limits for areas of arable land for particular crops generated by re-sowing constraints with the real past sowing plans (see Fig. 1). Since the crop succession requirements for wheat, barley and triticale are generally the same, we can cluster all these crops as 'cereals' when enumerating the re-sowing constraints. In the particular case of the South Moravian farm, the area restrictions arise for cereals, oilseed rape and potatoes, but as potatoes have not been cropped at the farm since 2000, we do not take them into account in enumerating the constraints. Applying the constraint formula (23), the limits for areas sowed by cereals, oilseed rape and both of them together were enumerated using the database of sowing plans realized by the farmer in the period 1995-2008.[1] Comparing the real sowing plans and constraint limits (see Fig. 1) we can

[1]Since oilseed rape may be resowed on the same piece of land not earlier than after 5 years, the first five years of the data set were used only as basis for the constraint calculation.

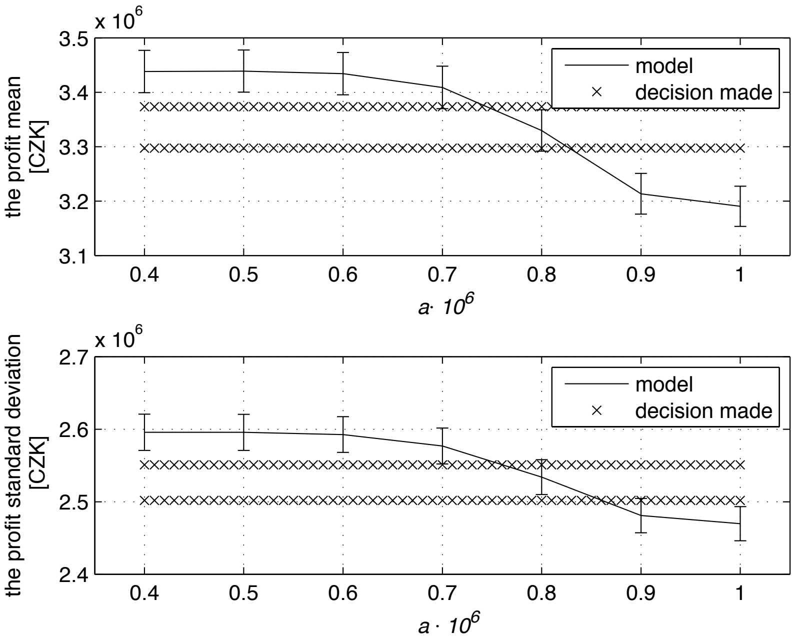

Figure 2: The confidence intervals for profit mean and standard deviation according to decider's risk attitude

see, that the farmer has been deciding in accordance with enumerated limits up to two exceptions in 2000 and 2007 where the farmer did not meet the crop succession requirements for cereals. Hence, the re-sowing constraint represents the true restrictions covered in the farmer's production planning except the cases when the succession requirements are disregarded by the farmer.

As we have seen above, the meaning of the resulting objective function in (17-22) differs from the profit. Hence, by validation, we should check whether the optimal sowing plan generates sufficient profit. The validation is performed using the Monte Carlo simulation and comparing the model solution profit characteristics with those resulting from the real sowing plan at the farm for 2009 (see Tab. 5). For the purpose of the harvests simulation, the natural yields (q_i) were described using the Beta distribution (see [21]) and the correlation between crop yields was considered using the Spearman rank correlation. Using the harvests simulation the profit characteristics (the 95% confidence interval for mean and standard deviation) were enumerated (the calculations were done in MATLAB). The results can be seen in Fig. 2 where, for the optimal sowing plan suggested by the model and for the real decision of the farmer, the particular confidence intervals are visualized for different risk aversion coefficients. We can see that, depending on the decision maker's risk attitude, the profit mean and standard deviation confidence intervals resulting from the modeled optimal sowing plan can be above or below those based on the real decision. There are two important conclusions following from the graph.

1. The profit characteristics of the model optimal solution are comparable to those obtained by the real sowing plan, which represents the current farmer's solution. This means that the model considering the randomness of the related processes provides a solution generating the profit that is comparable to the one obtained so far. The aim of using the decision support system is of course to find a sowing plan better than the one used so far by the farmer and, in this way, to increase the farmer's profit. Although the current real sowing plan seems to generate very good profit characteristics, one must take into account, that this sowing plan breaks the re-sowing constraint because, for the oil seed rape, there is only 131 ha of land available while the farmer sowed 230 ha of arable land by oilseed rape. The farmer assumes

that breaking the re-sowing constraint will not affect the harvests, which is not true from the long-term perspective. by the model such solution is infeasible, but if desired, the model can cover such exceptions.

2. The model reflects the risk as expected: the higher aversion to risk the lower the profit mean but also the lower the profit variance.

Conclusion

The model presented considers the harvests' randomness and all the restrictions on total costs, available land, available manure, minimal feasible harvests of feed crops and also crop succession requirements carried out for the Czech Republic region by incorporating the re-sowing constraints.

The results obtained from the model represent the areas of arable land cropped by particular crop plants and these areas fulfill the fundamental crop rotation rules while performing the expected profit at a sufficiently high level as we have shown by the Monte Carlo validation simulation. The re-sowing constraint itself is composed of a set of linear inequalities reflecting the crop succession requirements. Comparing the farmer's past sowing plan decisions and the constraint suggestions, we have verified the constraint functionality and relevance when confirming that the farmer keeps the upper area limits with some exceptions, when the advised crop succession rules were not kept. The model does not determine the "pattern" of the land, hence the farmer himself has to decide where the crops will be planted (it is, however, guaranteed by the model that such a configuration exists). The farmer can use the results of the model as true decision supporting information enabling him to correct or improve the up-coming sowing plan. The possibility of applying the model in farmer's practice is increased by the fact that the final model is solvable using an Excel spreadsheet which is commonly available at any farm.

References

[1] Bachinger, J., Zander, P.: ROTOR, a tool for generating and evaluating crop rotations for organic farming systems, EUROP J AGRONOMY, 26, (2007), 130-143.

[2] Benjamin, L.R. , Milne, A.E. , Parsons, D.J. , Cussans, J. , Lutman, P.J.W.: Using stochastic dynamic programming to support weed management decisions over a rotation WEED RES, 49 (2), (2009), 207-216.

[3] Bohle C., Maturana S., Vera J.: A robust optimization approach to wine grape harvesting scheduling. EUR J OPER RES (2009), doi: 10.1016/j.ejor.2008.12.003

[4] Bravo M., Gonzales I.: Applying stochastic goal programming: A case study on water use planning. Eur. J. Oper. Res. (2008), doi: 10.1016/j.ejor.2008.04.034

[5] Castellazzi, M.S. et al.: A systematic representation of crop rotations. AGR SYST, 97, (2008), 26–33.

[6] Detlefsen, N.K., Jensen, A.L.: Modelling optimal crop sequences using network flows, AGR SYST, 94 (2), (2007), 566-572.

[7] Dogliotti, S., Rossing, W.A.H., van Ittersum, M.K.: ROTAT, a tool for systematically generating crop rotations EUROPEAN JOURNAL OF AGRONOMY, 19 (2), (2003), 239-250.

[8] Dupacova, J.: Stochastic programming, MSMT, Praha, 1986, in Czech

[9] Freund, R.J.: The introduction of risk into a programming model. ECONOMETRICA 24 (1956), 253–263.

[10] Halachmi I. et al.:Validation of simulation model for robotic milkin barn design EUR J OPER RES, 134, (2001), 677–688.

[11] Klein Hanveled, W.K., Stegeman, A.W.: Crop succession requirements in agricultural production planning. EUR J OPER RES, 166, (2005), 406–429.

[12] Janová, J.: On treating the general constraints in agriculture and forestry optimization problems, 27th International Conference on Mathematical Methods in Economics, SEP 09-11, 2009 Kostelec nad Cernymi lesy, Czech Republic, 165–168.

[13] Janová, J., Ambrožová, P.: Optimization of production planning in Czech agriculture cooperative via linear programming. Acta univ. agric. et silvic. Mendel. Brun., 57 (6), (2009), 99–104.

[14] Jatoe, J.B.D., Yiridoe, E.K., Weersink, A., Clark, J.S.: Economic and environmental impacts of introducing land use policies and rotations on Prince Edward Island potato farms, LAND USE POLICY, 25 (3), (2008), 309-319.

[15] Maatman, A., Schweigman, C., Ruijs, A., van der Vlerk, M.H.: Modeling farmerŠs response to uncertain rain fall in Burkina Faso: A stochastic programming approach. OPER RES 50, (2002), 399–414.

[16] Myers, P., McIntosh, C.S., Patterson, P.E., Taylor, R.G., Hopkins, B.G.: Optimal crop rotation of Idaho potatoes AM J POTATO RES, 85 (3), (2008), 183-197.

[17] Paiva, R.P.O., Morabito, R.: An optimization model for the aggregate production planning of a Brazilian sugar and ethanol milling company, ANN OPER RES, 169 (1), (2009), 117 – 130.

[18] Parsons, D.J., Benjamin, L.R., Clarke, J., Ginsburg, D., Mayes, A., Milne, A.E., Wilkinson, D.J.: Weed Manager-A model-based decision support system for weed management in arable crops, COMPUT ELECTRON AGRIC, 65 (2), (2009), 155-167.

[19] dos Santos, L.M.R., Michelon P., Arenales M. N., Santos, R.H.S.: Crop rotation scheduling with adjacency constraints, ANN OPER RES, DOI 10.1007/s10479-008-0478-z

[20] Seppelt, R.: Regionalised optimum control problems for agroecosystem management, ECOL MODEL, 131 (2-3), (2000), 121-132.

[21] Spicka J., Boudny, J., Janotova J., The role of subsidies in managing the operating risk of agricultural enterprises, Agric. Econ.-Czech, 55(4), (2009), 169-179.

[22] Torkamani, J.: Using whole-farm modeling approach to assess prospective technologies under uncertainity. AGRIC. SYST. 85, (2005),138–154 .

[23] Vach, M., Javurek,M.: Rostlinná produkce s ohledem na agroekologická hlediska, Výzkumný ústav rostlinné výroby, v.v.i. 2008, ISBN 978-80-87011-58-4.