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# Time and frequency domain in the business cycle structure

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**Abstract**

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The paper deals with identification of cyclical behaviour of business cycle from time and frequency domain perspectives. Herewith, commonly used methods for obtaining growth business cycle are investigated – the first order difference, the unobserved component models, regression curves and filtration using Baxter-King and Christiano-Fitzgerald band-pass filters as well as Hodrick-Prescott high-pass filter. In the case of time domain analysis identification of cycle lengths is based on dating process of the growth business cycle. For this reason, methods such right and left variant of naive techniques as well as Bry-Boschan algorithm are applied. In the case frequency domain analysis of cyclical structure trough spectrum estimate via periodogram and autoregressive process with optimum lag are suggested. Results from both domain approaches are compared. On their bases recommendation for cyclical structure identification of growth business cycle of the transition economy type (the Czech Republic) are formulated. In the context of the time domain analysis evaluation of unity results of de-trending techniques from identification turning point points of view is attached. All analyses are done on the quarterly data of the gross domestic product, the total industry excluding construction, the gross capital formation in the period 1996/Q1-2008/Q4 and on the final consumption expenditure in the period 1995/Q1-2008/Q4.

**Key words**

spectrum, business cycle, transition economy, frequency domain, time domain

**JEL:** E32, C16, C5, C6

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## 1 Introduction

The issue of modelling the economic cycle is important especially in the context of the stabilizing function of economic policy. Identification of the different phases of cyclical fluctuations in economic activity, including the turning point is crucial not only for implementation of monetary and fiscal policy at the level of individual economies, but also in the broader context of economic integration process, especially from the perspective of the theory of optimum currency areas (the OCA). The rate of accuracy of identification of cyclical fluctuations in economic activity, including the turning points increases the monetary and fiscal policy implementation efficiency, especially symmetric and asymmetric shocks identification across the Eurozone.

Modelling fluctuations in economic activity through de-trending economic time series is common in the synchronisation of business cycles between the euro area member countries and newly acceding members. As shown in a meta-analysis by Fidrmuc and Korhonen (2006), this topic is at the forefront of economists' interests especially in the context of approach of the new Member States to monetary union. There are many methods for measuring the synchronisation of economic cycles. The most common is the unconditional correlation between the two countries in different time periods, the identification of delays various phases of business cycles, volatility of cyclical fluctuations in economic activity, stability and similarity of sudden and unexpected fluctuations in economic activity or shock response due to the level of the euro area as a whole or individual Member the euro area (Darvas and Szapáry, 2005). Using this approach, it is advisable to interpret the results obtained with respect to the nature of the economies under consideration. As Artis, Fidrmuc and Scharler (2008) found, the new EU Member States (and also the Czech Republic) higher correlation of business cycles is due to bilateral trade and financial flows. Basic list of indicators is completed by the methods of index of cyclical conformity, called Concordance Index (Harding and Pagan, 2006b), phase correlation (Koopman and Azevedo, 2003) or dynamic correlation (Croux, Forni and Reichlin, 2001). Individual studies provide the synchronisation of economic cycles, often significantly different results only because of different approaches to measuring their own synchronisation and usage of various types and data transformation, but mainly due to different methodology, modelling business cycles. This confirms the results of Fidrmuc and Korhonen (2006) show that estimation methods may have a significant impact on the assessment of alignment just using correlation coefficient.

The transformation process undergone by the Czech Republic after the fall of the communist regime and the subsequent trial, when the economy moves from central planning to a market system is an important factor in shaping and influencing the country's economy and its development (Kornai, 2006; Guriev a Zhuravskaya, 2009; Švejnar, 2002). This has significantly influenced the nature of macroeconomic indicators, and thus it pre-determined approach to analyzing the economic activity of the country modelled the economic cycle. Then, a more appropriate approach than the traditional concept of the economic cycle appears to be a growth cycle approach. While the various stages of growth of the business cycle are deviations from the long-term trend, classical concept of business cycle takes phases of the cycles as absolute declines' or increases in the real productions. In essence it is a problem of decomposition on a real economic output in trend and cyclical component. This trend is often replaced in discussion for potential output of economy and cyclical fluctuations in output gaps. Unfortunately, the long-term trend identified in the time series has little to do with the production of our economy. Parallels can be found only on the assumption that the economy in the long run on its potential, from which differs in the short term. Investment and other decisions of economic agents (f.e. labor fluctuation motives) are made rationally. People make mistakes, but they do not make the same mistakes over and over. Therefore, the economic system is in continual equilibrium and there no large asymmetries between rises and falls in output. That is, output growth is distributed symmetrically around its mean (or long term trend).

The above fact remains that the techniques used to estimate potential output and output gap are prone to uncertainties related to the processing of unknown features in the economy. Thus authors of empirical studies of the potential product often use de-trending techniques. For the purpose of this paper we assume the consensus growth business cycle with macroeconomic business cycle fluctuations. Identifying the cycle usually means his dating, thus setting turning points and the resolution phase of expansion and recession. Their knowledge is not only prior information subject to further application of selected methods for examining the economic cycle, but this is also important information for policy-makers in their decisions. This approach is, however, mainly responsible for analysis in time domain, which has long since been predominant. Requirements for this type of analysis are based on the needs of practical applications. A typical approach consisted of analyzing the correlation between selected indicators of economic activity (such as GDP, industrial) after filtering and the shocks in these variables. For the detail see work of Canova (1997, 1998), Ahumada and Garegnani (1990), Hodrick and Prescott (1981) or Bonenkamp (2001). The issue of dating the growth cycle is worked also in articles of Baxter and King (1999) and Harding and Pagan (2001, 2002 and 2006). Note that, like the range of the data set itself the choice of methods for growth business cycle obtaining may significantly affect the process of dating.

Dating the business cycle is based on the mathematical idea of setting points of the function extreme. In most approaches it simplify and generalize the requirements for a sequence of rising and decreasing values or addition on the growth cycle. Individual rules can not be applied mechanically, because the simplification of the mathematical approach, when applied to real data, it requires economic theory assumptions and analysis of graphical outputs. It is also appropriate because of the robustness of the results apply at least two approaches to the identification of turning points.

Evaluation and analysis of time series taking place in the time domain is unable to affect the frequency characteristics and frequency of time series. On the other hand, works in the frequency domain provides a deeper insight into the structure and the cyclical behaviour of time series in different time scales, as well as the development of time series decomposition in terms of periodic contribution. The economic cycle is typically defined in terms of period or frequency of adjustments. The spectral properties calculated by Fourier transformation (Hamilton, 1994; Schuster, 1898 and Jan, 1997) highlight a number of important differences in the cyclical properties of various measurements of the economic cycle. This approach is able to provide larger amounts of information, such as finding the frequency and length of cycles, which can be further analyzed in the study of the meaning of Schumpeter (1939) or Burda Wyplozse (2001). Then a combination of both approaches complement each other and comparison of the results is not only modern but also desirable.

Expanding the work from time to frequency domain can be found in the works of Guy St-Amant (1997) or Baxter-King (1999). In both works the authors focus attention on the band pass filter, high pass or low pass filters respectively. Frequency domain is also used for optimization of parameters when behavior of the spectrum or the ideal filter approximation is investigated. To increase the use of frequency analysis can be seen in the last decade. Many works using the techniques of frequency analysis was precisely in connection with the European Integration process, the countries examined its transitive economies in the euro area, the mutual alignment of the various aspects of development and the domestic economy. Among contemporary works can be included Paczinskieho and Wozniak (2007) calculated the spectrum using time-series autoregressive process through appropriate final order. Also work of Hallet and Richter (2004), who briefly considered the convergence of euro-area selected for estimating the spectrum of non-stationary processes.

Character of methods of the frequency analysis is purely descriptive and can not be used directly to predict future values. The age of a phase alone is not of much help in predicting the date of its end. There is a significant dynamics of the evolving business situation.

The motivation for the extension methods can be seen in the ever growing need for additional information, whether we consider the European view of co-movement, and hence the meaning of integration in the OCA, or in terms of linking the world's economies. Then, we can assess the global co-movements of economies worldwide, but also locally in terms of levels of state and representatives of the global total. A nice synthesis of empirical assessment of the international co-movement provided Mumtaz, and Surico Simonelli (2011), who evaluate co-movement in terms of globalization, namely at the level of world regions. The macroeconomic indicators represent complex economic system consist of many relations end elements. The rational explanation for business cycles fluctuations may be found in shifting demand and supply curves. This process leads to wage, investments and technology differentials between the different sectors and areas in the economy. Concurrently, the economic agents want to achieve high profit at low risk. They are constantly seeking profit, which appears. Therefore the agents diversify their investments according to their expectations. However, this diversification reduces the size of fluctuation (its amplitude). Subsequently, we can assume that there are many covered waves in the business cycle fluctutations. This assumtion is very important for the countries which joining the Eurozone as well. The issue of business cycle synchronization does not take place only on the basis of simple methods for measuring the synchronisation in the time domain, but also in the form of dynamic frequency correlations.

The proceeding in time in this direction is evident from studies of the development of writing on this topic (Fidrmuc and Korhonen 2006). The existence of a periodicity is also apparent when examining the time domain and not in the sense of conformity coordination itself, but also the development of alignment of alternating periods of larger and smaller synchronicity (Rinaldi-Larribe, 2008). The character of the methods of frequency analysis is purely descriptive and can not be used directly to predict future values. Although it is a powerful tool for the investigation of light cycles and relationships between time series, either the rear or expected.

The aim of the present paper is to assess the cyclical behavior of economic growth cycle of the Czech Republic from the perspective of time and frequency domains. In the time domain analysis the types of lengths of cycles identified on the basis of applications dating methods, namely the right and left versions of naive techniques and Bry-Boschan algorithm are done. In the frequency domain analysis for the cyclical behavior of the spectrum estimation method such periodogram and autoregressive process with optimized lag is investigated. In connection with the analysis in time domain is also analyzed the unity of the de-trending methods from the perspective of the identified turning points. The analysis is based on quarterly values of industrial production, investment and GDP in the period 1996-2008 and quarterly values of consumption in 1995-2008 all in the Czech Republic.

## 2 Material and Methods

In this paper, the nation's economic activity measured by the absolute values of the parameters will be transformed by natural logarithm and identified as Y. For the analysis of the economic cycle will be used additive decomposition. Thus, if we have time series $Y_t$, t = 1,..., n the regression relationship can be described by the equation in the form

$$Y_t = x_t + \varepsilon_t = g_t + c_t + s_t + \varepsilon_t, \quad t = 1,...,n, \qquad (1)$$

where $g_t$ denotes long-term trend or growth component, the cyclical component of $c_t$ arising from fluctuations of the business cycle, $s_t$ seasonal component (if the data are not adjusted for seasonality) and $\varepsilon_t$ is the irregular component, reflecting the piecemeal movement time series.

To remove the trend from the time series various methods can be used. In the group of de-trending techniques method such the first differences (FOD), deterministic models (regression line, LF; quadratic regression function, QF) or stochastic Unobserve component model (UC) belongs to. These methods can be generally described as statistical (Canova, 1998). As the representative of the statistical but non-parametric methods, will be accompanied by a non-parametric analysis of Gasser-Müller kernel estimation (Wand and Jones 1995). Another option provides filtering techniques, where Hodrick-Prescott (HP) filter, Baxter-King's (BK) filter or a Christian's filter-Fitzgerald (CF) (Baxter and King, 1999) is included.

In connection with the business cycle dating Bonenkamp (2001) discuss two the most common dating practices, the so-called naive techniques of dating the business cycle and complex Bry-Boschan approach. Canova (1999) uses a similar approach, which extends into two techniques. The first techniques that identifies the trough as a situation in which two consecutive quarters of decline in the growth cycle of economic growth is followed by one increase. Similarly, the peak is situations where two consecutive quarter of rising value are followed by one decrease. The second techniques selects the moment as trough of economic activity, respectively peak, if there have been at least two consecutive negative, respectively positive, spells in the cycle over a three quarter period. Denote the first techniques define above the left variant (NL). Marking the left is based on the requirement of two values before declining and one arise, rising values from the two top and one downward. If you accept that it is sufficient to identify the three extreme points, then the symmetric version of the techniques could also identify the extremes. Call this version of the right symmetric variant (NP) and the trough define a situation where the decline in economic growth cycle, followed by two consecutive quarters of rising. Similarly, peak as a situation where the growth cycle of economic growth followed by two consecutive quarters of decline.

More advanced techniques to generate turning points, which includes the following built-in, is called Bry-Boschan algorithm. This is an automated procedure that avoids the uncertainties of different interpretations of the basic principles of economic cycles. Bry-Boschan algorithm works such that the first was identified major cyclical movements, then established around the highs and lows and finally led to the search narrow turning points leading to the specification the moment of turning point. This algorithm takes into account the individual nature of time series. Comprehensive analysis of various statistical tools may even lead to a loss of stability of results over time. A more detaile description of this procedure can be found in Boschana and Brye (1971). Other advanced techniques can be for instance Markov-Switching model (Harding and Pagan, 2002a) or dating using non-parametric kernel estimation of time trend derivations (Poměnková, 2010a).

Applying analysis in the frequency domain, estimating the structure of the cyclic length is motivated by the identification of an effort of nested cycle in time series. Sample spectra can be estimated in several ways. In the case of nonparametric methods periodogram is a basic one, in the case of parametric methods can be spectrum estimated through autoregressive process (Hamilton, 1994). The basic premise is that the input time series is weakly stationary (Wooldridge, 2003).

Spectrum of time series $Y_t$ can be expressed by Fourier sum (De Jong, Chetan, 2007)

$$S_Y(\omega) = \frac{1}{2\pi}\sum_{j=-\infty}^{+\infty}\gamma_j e^{-i\omega j}, \qquad (2)$$

where $\gamma_j = \operatorname{cov}(Y_t, Y_{t+j}) = E(Y_t - \mu_t)(Y_{t+j} - \mu_{t+j})$ is autocovariance betwen $Y_t$ a $Y_{t+j}$, $i = \sqrt{-1}$. For the angular frequency ω in radians hold $\omega = 2\pi / n$, where n is a sample size.

Parametric spectrum estimation method using the autoregressive AR model can be calculated as (Proakis, 2002),

$$\hat{S}_Y(\omega) = \frac{\sigma_w^2}{\left|1 - \sum_{i=1}^{p} a_i e^{-j\omega in}\right|^2}, \qquad (3)$$

where $a_i$ are coefficients of the AR model of order p, n is the range of the data set (length of input time series). To estimate the parameters of the model AR(p) with optimum p value procedure of Yule-Walker method (Proakis, 2002) can be used. To optimize the delay in order autoregressive process can be used Akaike (AIC) or Schwartz-Cauchy (SC) information criterion (Seddidghi, 2000).

Nonparametric method for estimating the spectrum can be realized by periodogram. For the stationary process $Y_t$ with absolutly summable sequences of autocovariances for arbitrary ω we can construct value of sample periodogram in frequency analogously to the formula (1)

$$\hat{S}_Y(\omega) = \frac{1}{2\pi}\sum_{j=-n+1}^{n-1}\hat{\gamma}_j e^{-i\omega j}. \qquad (4)$$

Sample spectrum $S_Y(\omega)$ indicates the proportion of sample variance Y, which can be attributed to the cycle of frequency ω (Hamilton, 1994). .

Let $Y_1, Y_2, ..., Y_n$ be finite real sequence and let its values are presented in the form

$$Y_t = \mu + \sum_{j=1}^{M} \alpha_j \cdot \cos(\omega_j (t-1)) + \delta_j \cdot \sin(\omega_j (t-1)), \quad (5)$$

where $\alpha_i, \delta_i$ are random variables with zero mean value, that is $E(Y_t) = 0$ for all t. Sequence $\alpha_j, \delta_{j,} \ j = 1,.2,..,M$ are serially uncorrelated and mutually uncorelated. The variance $Y_t$ is then $E(Y_t^2) = \sum_{j=1}^{M} \sigma_j^2$. Thus, for this process, the portion of this variances Y that is due to cycles of frequency $\omega_j$ is given by $\sigma_j^2$ (Hamilton, 1994). If the frequences are ordered like $0 < \omega_1 < \omega_2 < ... < \omega_m < \pi$, the portion of the variance of Y is due to cycle of frequency less than or equal to $\omega_j$ is given by $\sigma_1^2 + \sigma_2^2 + ... + \sigma_j^2$. The $k^{th}$ autocovariance of Y is

$$E(Y_t Y_{t-k}) = \sum_{j=1}^{M} \sigma_j^2 \cdot \cos(\omega_j (k)). \quad (6)$$

The general result known as the spectral representation theorem states that any stationary process $Y_t$ can be expressed in the form (4). Estimates of the parameters $\hat{\alpha}_i, \hat{\delta}_i$ can be obtained using the OLS method. Thus, the periodogram can be written as

$$\hat{S}_Y(\omega_j) = \frac{1}{4\pi}\left(\hat{\alpha}_j^2 + \hat{\delta}_j^2\right), \ j=1,2,...,M. \quad (7)$$

**3 Data**

For the long term trend identification and the business cycle dating four macroeconomic indicators have been used which analyse main areas of economic activity in the Czech Republic. All these data are quarterly values, in millions of national currency, chain-linked volumes, reference year 2000 (including 'euro fixed' series for euro area countries) and are seasonally adjusted. Namely, the gross domestic product (the GDP), the total industry excluding construction (the industry) and the gross capital formation (the GCF); all these data are in the period 1996/Q1–2008/Q4. Also, the final consumption expenditure (the consumption) in the period 1995/Q1–2008/Q4 was used. All input values were sourced from the free Eurostat webpage and they were transformed into the natural logarithms.

**4 Application**

For a practical demonstration of described methods was done dating of the business cycle modelled on the indicators defined above. The growth cycle of these values was obtained by application of the following methods: method of first differences (FOD), unobserved components model (UC), de-trending using regression functions: linear function (LF), a quadratic function (QF), an estimate of Gasser-Müller (GM) estimate, the Hodrick- Prescott (HP), Baxter-King's (BK) filter and Christiano-Fitzgerald filter with pre-filtering using regression line (CF-LF) and quadratic regression functions (CF-QF).

In the case of the Hodrick-Prescott filter smoothing constant $\lambda$ = 1600 for quarterly values was taken (Hodrick and Prescott, 1980). Setting the frequency band pass filters (BK and CF filters) is based on the work of Guay and St-Amant (1997), when we recognize the highest frequency in the economic cycle length of 6 quarters and the lowest of 32 quarters. In the case of Baxter-King filter with respect to the sample size of date, selected length of the moving parts K = 7 (Baxter, King, 1999) was taken. As Christiano and Fitzgerald (2003) recommend, before applying Christiano-Fitzgerald filter should be removed from the original time series deterministic or another significant trend. In the case of value of investment, industry and consumption trends were removed using the regression line, in the case of GDP using a quadratic regression function, because the estimate showed better statistics than the estimated regression line.

For the business cycle dating process in the time domain, left and right variants of naive techniques and Bry-Boschanův algorithm were used. Dating methods have been applied to the values derived growth cycles by de-trending techniques mentioned above. At these growth cycles were also carried out assessment of the suitability of de-trending method trough to unity of obtained growth cycles in the sense of correlation (pair correlation) and in terms of unity of identified turning points, i.e. trough, peak, trough and peak. For more details see Poměnková (2010b).

**Table 1: Unity of growth cycles**

| Order | Peak | Trough | Correlation of cycles | Trough and peak unity |
|---|---|---|---|---|
| | **Industry** | | | |
| 1. | BK, HP; BK, QF | HP, LF | HP,QF | LF, QF |
| 2. | HP, LF, QF | QF, LF | CF-LF, BK | HP, LF |
| | GM, QF; | HP, GM; | HP, BK; | QF, GM |
| 3. | GM, HP | HP, LF, QF | QF, BK | |
| 4. | GM, BK | CF-LF, BK | | |
| | **GCF** | | | |
| 1. | HP, LF | HP, QF | HP, QF | LF, QF |
| | LF, QF; | LF, QF | CF-LF, BK | HP, LF |
| | CF-LF, HP; | | | |
| 2. | CF-LF, BK | | | |
| | HP, QF; | HP, LF | CF-LF, HP | HP, QF |
| 3. | CF-LF, LF | | | |
| | LF, GM; | HP, GM; BK, LF; CF- | LF, QF | CF-LF, QF |
| 4. | CF-LF, QF | LF, QF | | |
| | **GDP** | | | |
| 1. | CF-LF, LF | BK, HP; BK, QF | HP, BK | CF-LF, HP |
| | CF-LF, HP | CF-LF, HP; | CF-QF, BK | CF-LF, LF |
| 2. | | CF-QF, HP | | |
| | CF-LF, GM; | CF-LF, LF | QF, BK | CF-QF, HP |
| 3. | CF-QF,BK | | | |
| | CF-QF, QF; | HP, QF | QF, HP; | CF-QF, BK |
| 4. | CF-QF, HP | | CF-QF, HP | |
| | **Consumption** | | | |
| 1. | FOD, GM | LF, QF; LF, HP | HP, QF | LF, QF |
| 2. | HP, QF | HP, QF | CF-LF, BK | LF, HP |
| 3. | LF, QF; LF, HP | HP, LF, QF | CF-LF, HP | QF, HP |
| 4. | HP, GM; HP, LF, QF | CF-LF, BK | LF, QF; LF, HP | |

Source: Own calculation

As we can see from the results in the table 1, the method denoted as FOD, UC almost absent even in correlation nor in unity of trough, peak, trough and peak of the first four major matches. From a more detailed graphical analysis of the meaning derived growth cycles (FOD, UC) and their pairwise correlations can be detected, although methods that meet the requirements for application to the selected data, it does not seem suitable to handle the growth cycle of the Czech Republic. Despite this fact, we include this method in the following analysis to confirm the unsuitability of both methods. In the case of nonparametric de-trending using Gasser-Müller estimates that method is not recommended, rather, because during the construction of this estimate, there were the so-called edge effects (estimates with a greater bias to the end of the data file), which do not disappear even after the application of methods used for their treatment (Poměnková, 2005). Therefore, this method will not be included into the following analysis. Conversely, it appears appropriate to the method of band-pass filters (BK, CF) and high-pass Hodrick-Prescott filter, which provides very similar results to the de-trending using regression functions. This result is not surprising if we perceive the Hodrick-Prescott filter as the filter improved the regression line by adding a constraint to dynamic of growth component.

On the basis of the analysis of dating turning points were identified and the cycles length were calculated (Table 2). Note that according to Artis (2004) we consider the situation of the cycle from peak to peak, that is one phase of recession followed by a one phase of expansion.

**Table 2: Lengths of cycles in the number of years for various dating techniques**

| LF | | | QF | | | HPF | | | BK | | | CF-LF/QF | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| NL | NP | BB | NL | NP | BB | NL | NP | BB | NL | NP | BB | NL | NP | BB |
| **Industry** | | | | | | | | | | | | | | |
| 7,25 | 3,5 | 7,75 | 3,5 | 4 | 3,5 | 3,5 | 4 | 3,5 | 2 | 2 | 2 | 3,75 | 3,75 | 3,75 |
| | 7,75 | | 6 | 5,5 | 6 | 6 | 6 | 6 | 4 | 4 | 2,25 | 7,5 | 7,5 | 5,75 |
| | | | | | | | | | | | 1,75 | | | 1,75 |
| **GCF** | | | | | | | | | | | | | | |
| 6,25 | | 7,75 | 6,25 | 7,75 | 2 | 6,25 | 7,75 | 2 | 2,25 | 2,25 | 2,25 | 4,25 | 4,25 | 2 |
| | | 3,25 | 4,5 | 3 | 5,75 | 4,75 | | 2,5 | 3,5 | 3,5 | 1,5 | 3,5 | 3,5 | 2,25 |
| | | | | | 3 | | | 6,5 | | | 2 | 3,25 | 3,25 | 1,5 |
| **Consumption** | | | | | | | | | | | | | | |
| 7 | 4,75 | 6,5 | 6 | 6,5 | 6,5 | 11 | 6,5 | 6,5 | 4 | 4 | 4 | 6,75 | 6,75 | 4,25 |
| 4,75 | | 4,25 | 4 | 3,5 | 3,5 | | 3,5 | 3,5 | | | | 3,75 | 3,75 | 3,75 |
| **GDP** | | | | | | | | | | | | | | |
| | 11,5 | 7,5 | 3,75 | 3,75 | 3,75 | 4 | 4 | 4 | | | | 6,75 | 6,75 | 6,75 |
| | | 0 | 5,75 | 6,25 | 5,75 | | 7 | 7 | | | | 4 | 4 | 4 |

Source: Own calculation

In the case of the analysis in the frequency domain, it was first necessary to verify the stationarity of growth cycles (Table 3). Order delay (Lag) for Adjusted Dickey-Fuller (ADF) test was chosen using information criteria - Akaike (AIC), the Cauchy-Schwartz (SC) and Hannah-Quine (HQC) (Seddighi, Lawler and Catos, 2000), then by Jarque-Bera test of normality (Green, 2008) and Dickey-Fuller (DF) test for white noise (Seddighi, 2000) of obtained residue of ADF test.

**Table 3: Testing stationarity around zero constant**

| ADF | LF | QF | FOD/UC | HP | BK | CF-LF/QF | CF-HP |
|---|---|---|---|---|---|---|---|
| Industry | *** | *** | *** | *** | *** | *** | *** |
| Lag | 1 | 1 | 1 | 1 | 4 | 1 | 1 |
| GCF | *** | *** | *** | *** | *** | *** | *** |
| Lag | 13 | 1 | 1 | 1 | 3 | 1 | 1 |
| GDP | *** | *** | *** | *** | *** | *** | *** |
| Lag | 1 | 1 | 1 | 1 | 3 | 1 | 1 |
| Consumption | *** | *** | *** | *** | *** | *** | *** |
| Lag | 1 | 1 | 1 | 1 | 2 | 1 | 1 |

$H_0$: nonstationarity $e_t$, p-value > α
$H_1$: stationarity $e_t$, p-value < α, 1%(***), 5%(**), 10%(*)
Source: Own calculation

In estimating the spectrum we proceed as follows: i) the first parameter optimization and test of normality of residue for AR(p) process describing the input data; ii) estimation of model AR(p) with the optimized value of p, an estimate of the spectrum with methods for the AR(p) model will be proceed; iii) calculating an estimate of periodogram (sentence 1iii)); the so-called harmonic analysis will be applied; iv) establishment of significant lengths of cycles of the two used methods will be concluded.

To optimize the lag order of autoregressive process is used information criteria (AIC, SC, and HQC), in combination with testing for white noise obtained residue Dickey-Fuller (DF) test and the Jarque-Bera test of normality (Green, 2008). The optimum value will be considered as the p-value for which they received normal residue, the nature of white noise and that the criteria will be reported as the lowest value in relation to other lag orders. Given the above theoretical considerations will be elected rather high maximum lag, up to 20th order. As in the case of Baxter-King filter is a data loss due to moving the selected parts, the maximum value was chosen to optimize the AR process, 15th. Low levels of lag order are for to calculate the spectrum rather inappropriate. If the lag of the AR(p) process is a little, it can be used to calculate the spectrum only a small amount (exactly p) coefficients and an estimate of the spectrum can miss or suppress certain types of periodic components, which occur in the time series and by the effect of the influence of small p can remain hidden. Selecting too large a lag order during optimization process, however, also harbors a certain risk. When order value is too high, then sample size for estimation AR(p) process is going to be smaller. The resulting AR process, the calculation of the criteria and the actual statistical tests can then be influenced by small sample size and exposed to the risk of reduced quality of the final estimate. The advantage of higher lag order, having regard to information criteria, statistical tests and abovementioned difficulties is the fact that the greater the lag procedure, the greater number of obtained coefficients, may yield a more accurate picture of the periodic behavior of time series. Frequency, which need not to be visible in the case of small number of coefficients, may arise in the case of higher number of coefficients provided its existence in the time series. About the statistical significance of individual components of the periodic components we can decide using R. A. Fischer test. Optimal lag order value for AR(p) process for all indicators are in the following table (Table 4).

To estimate the coefficients of autoregressive process was due to higher optimal lag orders done using the Yule-Walker, which estimates the spectrum by autoregressive process of the autocorrelation function of the time series. In its calculation treats the parameters of autoregressive estimates also possible autocorrelation of residuals through a generalized regression.

**Table 4: Optimization of the parameter p of AR(p) procces of detrended values**

| p-optimum | *CF* | *HP* | *BK* | *LF* | *QF* | *FOD/UC* |
|---|---|---|---|---|---|---|
| Industry | *16* | *18* | *11* | *10* | *10* | *22* |
| GCF | *15* | *17* | *15* | *14* | *17* | *22* |
| GDP | *18* | *15* | *12* | *12* | *15* | *12* |
| Consumption | *19* | *18* | *12* | *21* | *18* | *20* |

Source: Own calculation

In calculating periodogram Fourier analysis for finding values of variability was. To calculate estimate of spectra the method AR(p) with optimum lag order (see Tab. IV) using the Yule-Walker method was applied. Reported results for both method were tested by R. A. Fischer test at 1%, 5% and 10% risk (Poměnková and Maršálek, 2010) (Table 5).

**Table 5: Estimation of the spectrum depending on the selected data and detrending method**

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Industry** | | | | | | | | | | | |
| | Periods | 12,75 | 6,38 | 4,25 | 3,19 | 2,55 | 2,13 | 1,82 | 1,59 | 1,42 | 0,91 |
| CF-LF | Per. | *** | *** | *** | *** | *** | | *** | | | |
| | AR | *** | *** | *** | *** | *** | *** | *** | *** | | |
| HP | Per. | | *** | ** | | | | | | | |
| | AR | | ** | ** | ** | | | | | | |
| LF | Per. | *** | *** | *** | | | | | | | |
| | AR | *** | *** | *** | ** | | | | | | |
| QF | Per. | | *** | ** | | | | | | | |
| | AR | | ** | *** | ** | | | | | | |
| FOD | Per. | | | | | | | | | | |
| | AR | | | | | | | | | | |
| | Periods | 9,25 | 4,63 | 3,08 | 2,31 | 1,85 | 1,54 | 1,32 | 1,16 | 1,03 | |
| BK | Per. | *** | *** | *** | *** | *** | | | | | |
| | AR | ** | *** | ** | ** | ** | ** | ** | | | |
| **GCF** | | | | | | | | | | | |
| | Periods | 12,75 | 6,38 | 4,25 | 3,19 | 2,55 | 2,13 | 1,82 | 1,59 | 1,42 | 0,91 |
| CF-LF | Per. | ** | *** | *** | *** | *** | *** | *** | *** | | |
| | AR | ** | *** | ** | ** | ** | ** | *** | ** | ** | |
| HP | Per. | | ** | | | | | * | | | |
| | AR | | ** | | | | | * | | | |
| LF | Per. | *** | *** | | | | | | | | |
| | AR | ** | ** | ** | | | ** | | | | |
| QF | Per. | | | | | | | | | | |
| | AR | | *** | | | | | | | | |
| FOD | Per. | | | | | | | | | | * |
| | AR | | | | | | | | | | ** |
| | Periods | 9,25 | 4,63 | 3,08 | 2,31 | 1,85 | 1,54 | 1,32 | 1,16 | 1,03 | |
| BK | Per. | | ** | *** | | *** | | | | | |
| | AR | ** | ** | ** | ** | ** | | | | | |
| **GDP** | | | | | | | | | | | |

| | Periods | 12,75 | 6,38 | 4,25 | 3,19 | 2,55 | 2,13 | 1,82 | 1,59 | 1,42 | 0,91 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| CF-QF | Per. | ** | *** | *** | ** | * | ** | ** | ** | | |
| | AR | ** | *** | *** | ** | | | | | | |
| HP | Per. | | *** | | | | | | | | |
| | AR | | *** | *** | | | | | | | |
| LF | Per. | *** | *** | *** | ** | | | | | | |
| | AR | *** | *** | *** | ** | ** | | | | | |
| QF | Per. | | | | | | | | | | |
| | AR | | *** | *** | | | | | | | |
| UC | Per. | ** | ** | | | | | | | | |
| | AR | | *** | | | | | | | | |
| | Periods | 9,25 | 4,63 | 3,08 | 2,31 | 1,85 | 1,54 | 1,32 | 1,16 | 1,03 | |
| BK | Per. | | *** | *** | | | | | | | |
| | AR | *** | *** | *** | ** | | | | | | |
| **Consumption** | | | | | | | | | | | |
| | Periods | 13,75 | 6,88 | 4,58 | 3,44 | 2,75 | 2,29 | 1,96 | 1,72 | 1,53 | 1,38 |
| CF-LF | Per. | ** | *** | ** | *** | *** | *** | ** | ** | ** | |
| | AR | *** | *** | *** | *** | *** | *** | *** | | | |
| HP | Per. | | *** | | *** | | ** | | | | |
| | AR | | *** | | *** | | | | | | |
| LF | Per. | *** | *** | *** | | | *** | | | | |
| | AR | ** | ** | ** | ** | ** | ** | | | | |
| QF | Per. | | *** | | *** | | ** | | | | |
| | AR | | *** | | *** | | | | | | |
| FOD | Per. | | | | | | | | | | |
| | AR | | | | | | | | | | |
| | Periods | 10,25 | 5,13 | 3,42 | 2,56 | 2,05 | 1,71 | 1,46 | | | |
| BK | Per. | ** | *** | *** | | *** | | | | | |
| | AR | | ** | *** | ** | ** | ** | | | | |

Source: Own calculation

Note: Statistically significant at a 5 %(***), %(**), 10 %(*), Periods are given in years. Per.– spectrum estimate using periodogram, AR – spectrum estimate using autoregressive AR(p) process

When results of spectral analysis are evaluated, thus an existing cycle length is indicated that length, which is confirmed by both methods, or has a high statistical significance (***). Furthermore, the lengths of the division cycles in length with very short duration of three years, with a short duration from three to five years, the median time duration from five to seven years and long cycles with duration of seven years.

The results of the estimation of lengths of cycles for industry (Table V), we can see the high unity of identified cycle lengths of the two methods in the case of long and medium lengths of cycles, and even cross-section of all de-trending methods. Growth cycles obtained by application of band-pass filters also identified a very short cycle length with duration of three years. In the case of investments (Table V) can be stated generally small number of identified types of cycles obtained for growth cycles de-trending using regression function or using the Hodrick-Prescott filter. In the case of band-pass filters, amount of identified types of cycles were increased. If we assess found cycles globally, both in cross section of de-trending methods and types of methods for estimating lengths of cycles (spectrum estimates), we see medium-term unity of cycles. Very good match occurs when the estimate of spectra is done using periodogram and using autoregressive process for band-pass filters, where except medium cycles can be identified short cycles as well.

The existence of a long cycle (12.75 years) when Christiano-Fitzgerald filter is applied is for debate. Given the set frequency range band-pass filter are likely to have been close to release so-called pass-band zone frequency components of the filters due to an approximation of ideal filter (Proakis, 2004 and Jan 1997). Instances of the same long cycle during de-trending using regression line, however, admits the existence of such a cycle. The same situation occurs in the case of other indicators under consideration. In all cases, the cycle length of 12.75 years is identified by both methods of spectrum estimation.

According to the results for value of GDP (table V) we can see that the estimates of lengths of cycles with periodogram were confirmed over the spectrum. Note that de-trending using quadratic regression functions for periodogram did not identify any type of cycle. In cross section of de-trending methods and types of methods for estimating nested loops can be stated as the existence of long cycles and cycles of medium length. In the case of the growth cycle obtained using de-trending by band-pass filters have been identified in addition to the short length of the cycles of three years. The analysis of consumption data (Table 5) showed a method of spectrum estimation through autoregressive process of consensus, identified types of cycles with lengths using periodogram. If we assess the value of consumption globally in cross section of de-trending methods, we see that consumption has nested cycles of long and medium distances as well as short cycle lengths. Even if the consumption values may likewise observe the situation of investment in applications Christiano-Fitzgerald filter not removing the longest cycle of band-pass filters.

Reviewing methods FOD and UC for obtaining growth cycle once again from the perspective of the results of spectral analysis, we see that in the case of industry and consumption have not identified any cycle length, in the case for investment cycle length of 0.9 years and, in the case for GDP cycle length of 6.38. Given the results of time domain, the author is inclined to conclude that both methods are for obtaining the growth cycle of the Czech Republic rather inappropriate.

## 5 Discussion

As stated by Canova (1999) or Bonenkamp (2001), de-trending affects the growth cycle values and, consequently, it’s dating. If we consider the robust results, the consensus of identified types of cycle lengths should not differ greatly in cross-section of de-trending techniques. While acknowledging the different nature of de-trending techniques, in all cases the aim of de-trending is to eliminate term trend component, while other variations and responses to shocks should be part of the short cyclical component. Then, there is every reason to think that, if existence of a certain type of cycle is occur using one method of de-trending, then another (good quality) de-trending technique the type of cycle should be also identified. Given that de-trending techniques are not identical, we can allow numerous variations, such as the statistical significance of identified cycle, or the existence of nearby cycles.

When evaluating the results of spectral analysis, the length of the cycle is denoted as existing, when is confirmed by both methods, or have a high statistical significance. The results are written in following table 6.

First examine the results of analysis in both time and frequency domain. As shown in Table 2, 5 and consequently in table 6, analysis in time and in frequency domain shows that the investigated time series include all types of defined lengths of cycles, i.e. cycles of very short, short, medium and long length. From this perspective, the approaches in both domains considered equally beneficial. On closer analysis, however, we find the following differences.

Evaluation of results in time domain is performed from perspective of de-trending function. We see that in the case of values of industry de-trended using regression line only naive techniques just allows the identification of short-cycle, in the case of investments it was Bry-Boschan algorithm. The growth cycle modelled on consumption values was not captured mid-cycle length by the right version of naive techniques and in the case of values of GDP by a long series of naive left.

**Table 6: The type of periods – time and frequency domain**

| Filter | | The length of the cycle (years) - **Industry** | | | | The length of the cycle (years) – **GCF** | | | |
|---|---|---|---|---|---|---|---|---|---|
| Filter | | very short till 3 y. | short 3–5 y. | middle 5–7 y. | long over 7 y. | very short till 3 y. | short 3–5 y. | middle 5–7 y. | long over 7 y. |
| LF | Time domain | | x | | x | | x | x | x |
| QF | Time domain | | x | x | | x | x | x | x |
| HP | Time domain | | x | x | | x | x | x | x |
| BK | Time domain | x | x | | | x | x | | |
| CF-LF | Time domain | x | x | x | x | x | x | | |
| | | till 3 y. | 3–5 y. | 5–7 y. | over 7 y. | till 3 y. | 3–5 y. | 5–7 y. | over 7 y. |
| LF | Frequency domain | | x | x | x | | | x | x |
| QF | Frequency domain | | x | x | | | | | |
| HP | Frequency domain | | x | x | | x | | x | |
| BK | Frequency domain | x | x | | x | x | x | | |
| CF-LF | Frequency domain | x | x | x | x | x | x | x | x |
| | | The length of the cycle (years) – **GDP** | | | | The length of the cycle (years) - **Consumption** | | | |
| Filter | | very short till 3 y. | short 3–5 y. | middle 5–7 y. | long over 7 y. | very short till 3 y. | short 3–5 y. | middle 5–7 y. | long over 7 y. |
| LF | Time domain | | | | x | | x | x | |
| QF | Time domain | | x | x | | | x | x | |
| HP | Time domain | | x | x | | | x | x | x |
| BK | Time domain | | | | | | x | | |
| CF-LF | Time domain | | x | x | | | x | x | |
| | | till 3 y. | 3–5 y. | 5–7 y. | over 7 y. | till 3 y. | 3–5 y. | 5–7 y. | over 7 y. |
| LF | Frequency domain | | x | x | x | x | x | x | x |
| QF | Frequency domain | | | | | | x | x | |
| HP | Frequency domain | | | x | | | x | x | |
| BK | Frequency domain | | x | | | x | x | x | |
| CF-QF/LF | Frequency domain | | x | x | x | x | x | x | x |

Note: x denotes the existence of type of periodicity
Source: Own calculation

In the case of de-trending using the Hodrick-Prescott filter was not in the values of investments by the right version of naive techniques identified cycle of mid, short and very short length, in the case of the left variant of naive techniques a very short cycle length and in the case of Bry-Boschan algorithm short cycle length. In the case of consumption values left variant of naive techniques did not allow finding the short cycle length, in the case of GDP mean cycle length. De-trending using Baxter-King's filter and post-dating by Bry-Boschan algorithm against the values of industry have not found a short cycle length; de-trending using Christiano-Fitzgerald filter have not found long cycle lengths. If de-trending using quadratic functions was done, we can see identical results (the same type of cycle lengths) with all the dating methods. Identical results were also achieved in the Hodrick-

Prescott filtered values of industry, and Baxter-King's and Christiano-Fitzgerald filters for the values of investment, consumption and GDP.

From the above interpretation of the results implies that the analysis in time domain in terms of the results and subsequent interpretation places increased demands on attention and a number of methods used to describe the best indicator of a cyclic structure under consideration. Confirmation of these results should be completed either the results of the frequency domain, or other methods of dating. The indisputable fact remains that we describe in detail the cyclical behavior of the Czech Republic in the time domain, it is necessary to use several dating techniques and techniques to obtain the growth cycle. Such an analysis is rather robust, but time-demanding.

The results from analysis in the frequency domain also cover a wide range of lengths of cycles. We can observe that for the values of industry, investment and GDP were identified in the same cycle length, where the values of consumption frequency analysis revealed the existence of extra very short cycles. Accordingly, it can be concluded that the use of Christiano-Fitzgerald band-pass filter leads in case the value of industry, investment and GDP to obtain the same length as the total cycle analysis in time domain, whereas for consumption values usage of this filter leads to identification of addition existence of very short cycle. In the analysis of both domains is then possible for the values of industry and investment consistently observed that the methods de-trending using regression function and the Hodrick-Prescott filter function in terms of performance, as band-pass filters. A clear advantage is less time demand to obtain results, and greater accuracy; the results of the spectrum estimation using periodogram and autoregressive process showed a high compliance. The analysis shows that all considered indicators show the short, medium and long nested cycles. The values of the growth cycles of investment and consumption additionally include an existence of very short cycles.

If we take the distribution of cycle types as stated by Schumpeter (1939), cycles can be divided into so-called Kitchin’s cycles of length 3-5 years due to inventory movements, Juglar’s cycles of a length of 7-11 years due to investment in machinery and equipment and infrastructure Kuznets’ cycles of a length of 15-25 years associated with the growth of population and labor resources. In connection with the available data it was possible to identify Kitchin’s and Juglar’s cycles.

The findings thus suggest that the industrial production of the Czech Republic in terms of cyclical trends indicated by the movement of stocks. Recognizing the existence of long cycles, which showed a growth cycle based on the values of industrial production de-trended using Baxter-King's filter, then they are probably the source of long-term investment in machinery, equipment and technology.

Empirical knowledge about the business cycle also dealt Czesaný (2006). His study shows that the Czech economy has undergone in the period 1990-2003 two economic cycles, and one lasting 7 years

(1990-1997) and second in duration 6 years (1997-2003). The finding of these cycles is based on the economic analysis of the situation in the Czech Republic and can be considered a comprehensive assessment of the economic indicators of supply and demand sides. If we compare this with the results of the analysis of cyclical behavior, we see that the cycle lengths have also been identified and labeled as medium-term cycles.

## 6 Conclusion

The article dealt with the assessment of the cyclical behavior of economic growth cycle of the Czech Republic from the perspective of time and frequency domains in the context of the methods currently used to identify the economic growth cycle. When working in the time domain, the types of lengths of cycles on the basis of dating were identified. For the dating application of left and right variants of naive techniques and Bry-Boschan algorithm were done. In the frequency domain analysis of cyclic behavior the methods of spectrum estimation were used, namely periodogram and autoregressive process with optimized lag order. In connection with the analysis in time domain also the match of de-trending methods from the perspective of the identified turning points were analyzed.

The results in the time domain shows that by dating the growth cycle in this domain can not through the setting of turning points be reveal the existence of nested cycles. On the dating process of the cycle, may be able to determine what types of cycles (where there are more than one cycle) of the analyzed indicators exists, however, about the cyclical behavior we will not see too much. In this case, if the interest of analyst is focused on the structure and nature of cyclical behavior, it is advisable to use spectral analysis. Performing a spectral analysis periodogram and spectrum estimation using the autoregressive process with optimized lag order was used. Due to the length of time series and lag order of the autoregressive process Yule-Walker methods for estimating coefficients of autoregressive process was applied. If we assess the methods of spectral analysis used for cyclic behavior description, we observed good agreement between the results of both methods in relation to the method used for de-trending. The analysis allows us to do the analysis beyond the cyclical behavior of indicators to assess secondary the suitability of de-trending techniques. It was confirmed the unsuitability the FOD and UC methods respectively, such de-trending methods for obtaining the economic growth cycle in the Czech Republic.

From the empirical point of view we can say that all indicators show short, medium and long nested cycles. The values of the growth cycles of industry, investment and consumption additionally include a very short nested cycles. Comparison of the results (Table 5) shows that the identification of lengths of cycles based on the dating of the economic growth cycle in the Czech Republic in the time domain does not allow detection of nested cycles. With the addition of spectral analysis, we find that the consumption was in the data structure found a very short cycle, in the case of GDP and consumption the spectral analysis confirmed the existence of long cycles. In the above mentioned two indicators, the long cycles have been identified in the time domain only for one growth cycle.

Accordingly, it can be concluded that the use of Christiano-Fitzgerald band-pass filter resulted in case the value of industry, investment and GDP to obtain the same length as the total cycle analysis in time domain, where the consumption values for the above-mentioned identification addition and the existence of very short cycle. Then, in analyses in both domains is possible, for the values of industry and investment, consistently observed that the methods for de-trending using regression function and the Hodrick-Prescott filter work in terms of performance as band-pass filters. A clear advantage is less time demand to obtain results, and greater accuracy; the results of spectrum estimation using periodogram and autoregressive process showed a high unity. The analysis shows that all considered indicators show short and medium cycles in the time domain; short, medium and long nested cycles in the frequency domain. The values of the growth cycles of industry, investment and consumption additionally include existence of very short nested cycles. Both methodological approach is therefore appropriate to use in parallel and results in the mutual context.

This result is consistent with the nature of the indicators with GDP as an indicator of aggregate economic activity across the country includes short-term fluctuations in economic activity. Industry as an indicator is although focused on the sector of national economy, but in the Czech Republic reflects the overall nature of the economy. The GDP indicator can be replaced by indicator industry at least in respect of the identification cycle length. In the case of investment analysis confirmed the significant impact on GDP. Length of cyclic variation is related to the high volatility of the investment cycle, which is typical for the Czech Republic. In the case of consumption, it is clear that the demand side of the economy responds quickly to changes and shocks in the economy.

## References


Anděl, J. (1976), *Statistická analýza časových řad*, SNTL, Praha, 271 pp.

Artis, M., Marcellino, M., Proietti, T. (2004), Characterising the Business Cycles for Accession countries. CEPR-EABCN Conference of Business Cycle and Acceding Countries, Vienna.

Artis, M. J., Fidrmuc, J., Scharler, J. (2008), The transmission of business cycles, *Economic of Transition*, Vol. 16, No. 3, pp. 559-582.

Bonenkamp, J., Jacobs, J., Kuper, G. H. (2001), Measuring Business Cycles in the Netherlands, 1815 – 1913: A comparison of Business Cycle Dating Methods. *SOM Research Report*, No. 01C25 , Systems, Organisation and Management, Groningen. Uversity of Groningen [online].

Baxter, R, King, R. G. (1999), Measuring Business Cycles: Approximate Band – Pass Filters for Economic Time Series. *Review of Economic and Statistics*, vol. 81, no. 4, pp. 575-593.

Bry, G., Boschan, C. (1971), Cyclical Analysisof Time Series: Selected Procedures and Computer Programs, *Technical Paper* 20, National Bureau of Economic Researche, New York.

Burda, M., Wyplozs, C. (2001), *Macroeconomics*. A European text. 3rd ed. Oxford: Oxford University Press, 572 pp., ISBN 0-19-877650-0.

Burns, A. F., Mitchell, W. C. (1946), *Measuing Business Cycles*. New York, National Bureau of Economic Research, 590 pp., ISBN: 0-870-14085-3.

Canova, F. (1998), De-trending and business cycle facts, *Journal of monetary Economic*, vol. 41, pp. 533-540.

Canova, F. (1999), Does De-trending Matter for the Determination of the Reference Cycle nad Selection of Turniny Points?, *The Economic Journal*, Vol. 109, No. 452 (Jan., 1999), pp. 126-150.

Czesaný, S., (2006), *Hospodářský cyklus – teorie, monitorování, analýza, prognóza*. Linde Praha, Praha, 199 s., ISBN 80-7201-576-1.

Eurostat [on-line] (2009), [cit. 2009-03-18] National Accounts (including GDP)} Available at <http://epp.eurostat.ec.europa.eu/portal/page/portal/\\national_{}accounts/data/

database}>.

Green, W. H. (1997), *Econometric Analyses*. London: Prentice – Hall, 1997. 1076 s. ISBN 0-13-7246659-5.

Guriev, S., Zhuravskaya, E. (2009), (Un)Happiness in Transition, *Journal of Economic Perspectives*, Vol. 23, No. 2, pp. 143-168.

Guay, A. St-Amant, P. (1997), Do the Hodrick-Prescott and Baxter-king Filters Provide a Good Approximation of Business Cycles? Université a Québec á Montreál, *Working paper* No. 53.

Halleth, A. H., Richter, C. (2004), A Time-frequency Analysis of the Coherences of the US Business Cycle and the European Business Cycle, *CEPR Discussion Paper* no. 4751, London, Centre for Economic Policy Research.

Harding, D., Pagan, A. (2002a), A comparison of Two Business Cycles Dating Methods. *Journal of Economic Dynamics and Control*. vol. 27, pp. 1681-1690.

Harding, D., Pagan, A. (2002b), Dissecting the Cycle: A Methodological Investigation. *Journal of Monetary Economics*, vol 49., pp. 365-381.

Harding, D., Pagan, A. (2006), Measurement of Business Cycles, *Research paper* number 966, Melbourne, 2006, ISSN 0819-2642 .

Härdle, W. (1990), *Applied Nonparametric Regression*. Cambridge University Press, 349 pp., ISBN 10: 0521429501 .

Hamilton, J. D. (1994), *Time Series Analysis*. Princeton University Press, 799 pp., ISBN -10:0-691-04289-6.

Hodrick, R. J., Prescott, E. C. (1980), Post-war U.S. Business Cycles: An Empirical Investigation, Mimeo, Carnegie-Mellon University, Pitsburgh, PA. 1980, 24 pp.

Fidrmuc, J., Korhonen, I. (2006), Meta-analysis of the business cycle correlation between the euro area and CEECs, *Journal of Comparative Economics*, Vol. 34, pp. 518-537.

Christano, L. J., Fitzgerald, T. J. (1999), The band pass filter, *Working paper* 9906, Federal reserve bank of Cleveland.

Iacobucci, A. (2003), Spectral Analysis for Economic Time Series', OFCE *Working Paper* No. 2003-07.

Ján, J. (1997), *Číslicová filtrace, analýza a restaurace signálů*, VUT v Brně, VUTIUM, 427 s., ISBN 80-214-0816-2.

Kornai, J. (2006), The Great transformation of Central Eastern Europe, Success and disappointment*,* *Economic of Transition*, Vol. 14, n.2, pp. 207-244.

Larribe-Rinaldi, J.-M. (2008), Is economic convergence in New Member States sufficient for and adoption of the Euro*? The European Journal of Comparative Economics*, Vol. 5, n. 2, pp. 133-154 ISSN 1722-4667.

Mumtaz, H., Simonelli, S., Surico, P. (2011), International comovements, business cycle and inflation: A historical perspective, *Review of Economic Dynamics*, Vol. 14, pp. 176-198.

Poměnková, J. (2005), Some aspects of regression function smoohting (1st ed.). Ostrava: PhD-thesis.

Poměnková, J. (2010a), An alternative Approach to the Dating of Business Cycle: Nonparametric Kernel Estimation. *Prague Economic Papers*, No. 3, pp. 251 – 27, ISSN 1210-0455.

Poměnková, J. (2010b), Cyclicality of industrial production in the context of time and frequency domain, *Acta Universitatis Agriculturae et Silviculturae Mendelianae Brunensis*, Vol. LVIII, No. 6, pp. 355-368 , ISSN 1211-8516.

Poměnková, J., Maršálek, R. (2010), Testování významnosti periodicity průmyslové výroby pomocí R. A. Fisherova testu. *Acta Universitatis Agriculturae et Silviculturae Mendelianae Brunensis*.2010. sv. LVIII, č. 3, s. 189-195. ISSN 1211-8516.

Proakis, J. G., Rader, Ch. M., Ling, F.L., Nikias, Ch.L., Moonen, M., Proudler, J.K. (2002), *Algorithms for Statistical Signal Processing*, Prentice Hall, ISBN 0-13-062219-2.

Seddidghi, H. R., Lawler, K. A., Katos., A. V., (2000), *Econometrics. A practical approach*. New York 2000, pp. 262 – 287.

Švejnar, J. (2002), Transition economies: Performance and challenges, *Journal of Economic Perspectives*, Vol. 16, No. 1, pp. 3-28.

Schumpeter, J. A. (1939), *Business Cycles. A theoretical, Historical and Statistical Analysis of the Capitalist Process*, New York, 461 pp.

Wand, M. P., Jones, M. S. (1995), *Kernel Smoothing* (1st ed.). Chapman & Hall, London.

Woolddridge, J. M. (2003), *Introductory Econometrics: A modern approach*. Ohio 2003, 863 pp., ISBN 0-324-11364-1.

Wozniak, P., Paczynski (2007), Business Cycle Coherence between Euro Area and the EU New Member State: a Time-Frequency Analysis. Center for Economic Research and Graduate Education and the Economics Institute (CERGE-EI), Prague.